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Applying Piaget's Theory of Cognitive Development to Mathematics Instruction

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This paper is based on a presentation given at National Council of Teachers of Mathematics (NCTM) in 2005 in Anaheim, California. It explicates the developmental stages of the child as posited by Piaget. The author then ties each of the stages to developmentally appropriate mathematics instruction. The implications in terms of not imposing unfamiliar ideas on the child and importance of peer interaction are highlighted.

Introduction

Jean Piaget's work on children's cognitive development, specifically with quantitative concepts, has garnered much attention within the field of education. Piaget explored children's cognitive development to study his primary interest in genetic epistemology. Upon completion of his doctorate, he became intrigued with the processes by which children achieved their answers; he used conversation as a means to probe children's thinking based on experimental procedures used in psychiatric questioning.

One contribution of Piagetian theory concerns the developmental stages of children's cognition. His work on children's quantitative development has provided mathematics educators with crucial insights into how children learn mathematical concepts and ideas. This article describes stages of cognitive development with an emphasis on their importance to mathematical development and provides suggestions for planning mathematics instruction.

The approach of this article will be to provide a brief discussion of Piaget's underlying assumptions regarding the stages of development. Each stage will be described and characterized, highlighting the stage-appropriate mathematics techniques that help lay a solid foundation for future mathematics learning. The conclusion will incorporate general implications of the knowledge of stages of development for mathematics instruction.

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Underlying Assumptions

Piaget believed that the development of a child occurs through a continuous transformation of thought processes. A developmental stage consists of a period of months or years when certain development takes place. Although students are usually grouped by chronological age, their development levels may differ significantly (Weinert & Helmke, 1998), as well as the rate at which individual children pass through each stage. This difference may depend on maturity, experience, culture, and the ability of the child (Papila & Olds, 1996). According to Berk (1997), Piaget believed that children develop steadily and gradually throughout the varying stages and that the experiences in one stage form the foundations for movement to the next. All people pass through each stage before starting the next one; no one skips any stage. This implies older children, and even adults, who have not passed through later stages process information in ways that are characteristic of young children at the same developmental stage (Eggen & Kauchak, 2000).

Stages of Cognitive Development

Piaget has identified four primary stages of development: sensorimotor, preoperational, concrete operational, and formal operational.

Sensorimotor Stage

In the sensorimotor stage, an infant's mental and cognitive attributes develop from birth until the appearance of language. This stage is characterized by the progressive acquisition of object permanence in which the child becomes able to find objects after they have been displaced, even if the objects have been taken out of his field of vision. For example, Piaget's experiments at this stage include hiding an object under a pillow to see if the baby finds the object.

An additional characteristic of children at this stage is their ability to link numbers to objects (Piaget, 1977) (e.g., one dog, two cats, three pigs, four hippos). To develop the mathematical capability of a child in this stage, the child's ability might be enhanced if he is allowed ample opportunity to act on the environment in unrestricted (but safe) ways in order to start building concepts (Martin, 2000). Evidence suggests that children at the sensorimotor stage have some understanding of the concepts of numbers and counting (Fuson, 1988). Educators of children in this stage of development should lay a solid mathematical foundation by providing activities that incorporate counting and thus enhance children's conceptual development of number. For example, teachers and parents can help children count their fingers, toys, and candies. Questions such as "Who has more?" or "Are there enough?" could be a part of the daily lives of children as young as two or three years of age.

Another activity that could enhance the mathematical development of children at this stage connects mathematics and literature. There is a plethora of children's books that embed mathematical content. (See Appendix A for a non-exhaustive list of children's books incorporating mathematical concepts and ideas.) A recommendation would be that these books include pictorial illustrations. Because children at this stage can link numbers to objects, learners can benefit from seeing pictures of objects and their respective numbers simultaneously. Along with the mathematical benefits, children's books can contribute to the development of their reading skills and comprehension.

Preoperational Stage

The characteristics of this stage include an increase in language ability (with over-generalizations), symbolic thought, egocentric perspective, and limited logic. In this second stage, children should engage with problem-solving tasks that incorporate available materials such as blocks, sand, and water. While the child is working with a problem, the teacher should elicit conversation from the child. The verbalization of the child, as well as his actions on the materials, gives a basis that permits the teacher to infer the mechanisms of the child's thought processes.

There is lack of logic associated with this stage of development; rational thought makes little appearance. The child links together unrelated events, sees objects as possessing life, does not understand point-of-view, and cannot reverse operations. For example, a child at this stage who understands that adding four to five yields nine cannot yet perform the reverse operation of taking four from nine.

Children's perceptions in this stage of development are generally restricted to one aspect or dimension of an object at the expense of the other aspects. For example, Piaget tested the concept of conservation by pouring the same amount of liquid into two similar containers. When the liquid from one container is poured into a third, wider container, the level is lower and the child thinks there is less liquid in the third container. Thus the child is using one dimension, height, as the basis for his judgment of another dimension, volume.

Teaching students in this stage of development should employ effective questioning about characterizing objects. For example, when students investigate geometric shapes, a teacher could ask students to group the shapes according to similar characteristics. Questions following the investigation could include, "How did you decide where each object belonged? Are there other ways to group these together?" Engaging in discussion or interactions with the children may engender the children's discovery of the variety of ways to group objects, thus helping the children think about the quantities in novel ways (Thompson, 1990).

Concrete Operations Stage

The third stage is characterized by remarkable cognitive growth, when children's development of language and acquisition of basic skills accelerate dramatically. Children at this stage utilize their senses in order to *know*; they can now consider two or three dimensions simultaneously instead of successively. For example, in the liquids experiment, if the child notices the lowered level of the liquid, he also notices the dish is wider, seeing both dimensions at the same time. Additionally, seriation and classification are the two logical operations that develop during this stage (Piaget, 1977) and both are essential for understanding number concepts. Seriation is the ability to order objects according to increasing or decreasing length, weight, or volume. On the other hand, classification involves grouping objects on the basis of a common characteristic.

According to Burns & Silbey (2000), "hands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage" (p. 55). The importance of hands-on activities cannot be overemphasized at this stage. These activities provide students an avenue to make abstract ideas concrete, allowing them to get their hands on mathematical ideas and concepts as useful tools for solving problems. Because concrete experiences are needed, teachers might use manipulatives with their students to explore concepts such as place value and arithmetical operations. Existing manipulative materials include: pattern blocks, Cuisenaire rods, algebra tiles, algebra cubes, geoboards, tangrams, counters, dice, and spinners. However, teachers are not limited to commercial materials, they can also use convenient materials in activities such as paper folding and cutting. As students use the materials, they acquire experiences that help lay the foundation for more advanced mathematical thinking. Furthermore, students' use of materials helps to build their mathematical confidence by giving them a way to test and confirm their reasoning.

One of the important challenges in mathematics teaching is to help students make connections between the mathematics concepts and the activity. Children may not automatically make connections between the work they do with manipulative materials and the corresponding abstract mathematics: "children tend to think that the manipulations they do with models are one method for finding a solution and pencil-and-paper math is entirely separate" (Burns & Silbey, 2000, p. 60). For example, it may be difficult for children to conceptualize how a four by six inch rectangle built with wooden tiles relates to four multiplied by six, or four groups of six. Teachers could help students make connections by showing how the rectangles can be separated into four rows of six tiles each and by demonstrating how the rectangle is another representation of four groups of six.

Providing various mathematical representations acknowledges the uniqueness of students and provides multiple paths for making ideas meaningful. Engendering opportunities for students to present mathematical solutions in multiple ways (e.g., symbols, graphs, tables, and words) is one tool for cognitive development in this stage. Eggen & Kauchak (2000) noted that while a specific way of representing an idea is meaningful to some students, a different representation might be more meaningful to others.

Formal Operations Stage

The child at this stage is capable of forming hypotheses and deducing possible consequences, allowing the child to construct his own mathematics. Furthermore, the child typically begins to develop abstract thought patterns where reasoning is executed using pure symbols without the necessity of perceptive data. For example, the formal operational learner can solve $x + 2x = 9$ without having to refer to a concrete situation presented by the teacher, such as, "Tony ate a certain number of candies. His sister ate twice as many. Together they ate nine. How many did Tony eat?" Reasoning skills within this stage refer to the mental process involved in the generalizing and evaluating of logical arguments (Anderson, 1990) and include clarification, inference, evaluation, and application.

Clarification. Clarification requires students to identify and analyze elements of a problem, allowing them to decipher the information needed in solving a problem. By encouraging students to extract relevant information from a problem statement, teachers can help students enhance their mathematical understanding.

Inference. Students at this stage are developmentally ready to make inductive and deductive inferences in mathematics. Deductive inferences involve reasoning from general concepts to specific instances. On the other hand, inductive inferences are based on extracting similarities and differences among specific objects and events and arriving at generalizations.

Evaluation. Evaluation involves using criteria to judge the adequacy of a problem solution. For example, the student can follow a predetermined rubric to judge the correctness of his solution to a problem. Evaluation leads to formulating hypotheses about future events, assuming one's problem solving is correct thus far.

Application. Application involves students connecting mathematical concepts to real-life situations. For example, the student could apply his knowledge of rational equations to the following situation: "You can clean your house in 4 hours. Your sister can clean it in 6 hours. How long will it take you to clean the house, working together?"

Implications of Piaget's Theory

Critics of Piaget's work argue that his proposed theory does not offer a complete description of cognitive development (Eggen & Kauchak, 2000). For example, Piaget is criticized for underestimating the abilities of young children. Abstract directions and requirements may cause young children to fail at tasks they can do under simpler conditions (Gelman, Meck, & Merkin, 1986). Piaget has also been criticized for overestimating the abilities of older learners, having implications for both learners and teachers. For example, middle school teachers interpreting Piaget's work may assume that their students can always think logically in the abstract, yet this is often not the case (Eggen & Kauchak, 2000).

Although not possible to teach cognitive development explicitly, research has demonstrated that it can be accelerated (Zimmerman & Whitehurst, 1979). Piaget believed that the amount of time each child spends in each stage varies by environment (Kamii, 1982). All students in a class are not necessarily operating at the same level. Teachers could benefit from understanding the levels at which their students are functioning and should try to ascertain their students' cognitive levels to adjust their teaching accordingly. By emphasizing methods of reasoning, the teacher provides critical direction so that the child can discover concepts through investigation. The child should be encouraged to self-check, approximate, reflect and reason while the teacher studies the child's work to better understand his thinking (Piaget, 1970).

The numbers and quantities used to teach the children number should be meaningful to them. Various situations can be set up that encourage mathematical reasoning. For example, a child may be asked to bring enough cups for everybody in the class, without being explicitly told to count. This will require them to compare the number of people to the number of cups needed. Other examples include dividing objects among a group fairly, keeping classroom records like attendance, and voting to make class decisions.

Games are also a good way to acquire understanding of mathematical principles (Kamii, 1982). For example, the game of musical chairs requires coordination between the set of children and the set of chairs. Scorekeeping in marbles and bowling requires comparison of quantities and simple arithmetical operations. Comparisons of quantities are required in a guessing game where one child chooses a number between one and ten and another attempts to determine it, being told if his guesses are too high or too low.

Summary

As children develop, they progress through stages characterized by unique ways of understanding the world. During the sensorimotor stage, young children develop eye-hand coordination schemes and object permanence. The preoperational stage includes growth of symbolic thought, as evidenced by the increased use of language. During the concrete operational stage, children can perform basic operations such as classification and serial ordering of concrete objects. In the final stage, formal operations, students develop the ability to think abstractly and metacognitively, as well as reason hypothetically. This article articulated these stages in light of mathematics instruction. In general, the knowledge of Piaget's stages helps the teacher understand the cognitive development of the child as the teacher plans stage-appropriate activities to keep students active.

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Appendix A: Children's Literature Incorporating Mathematical Concepts and Ideas

Anno, M. (1982). *Anno's counting house.* New York: Philomel Books.

Anno, M. (1994). *Anno's magic seeds.* New York: Philomel Books.

Anno, M., & Anno, M. (1983). *Anno's mysterious multiplying jar.* New York: Philomel Books.

Ash, R. (1996). *Incredible comparisons.* New York: Dorling Kindersley.

Briggs, R. (1970). *Jim and the beanstalk.* New York: Coward–McCann.

Carle, E. (1969). *The very hungry caterpillar.* New York: Putnam.

Chalmers, M. (1986). *Six dogs, twenty-three cats, forty-five mice, and one hundred sixty spiders.* New York: Harper Collins.

Chwast, S. (1993). *The twelve circus rings.* San Diego, CA: Gulliver Books, Harcourt Brace Jovanovich.

Clement, R. (1991). *Counting on Frank.* Milwaukee: Gareth Stevens Children's Book.

Cushman, R. (1991). Do *you wanna bet? Your chance to find out about probability.* New York: Clarion Books.

Dee, R. (1988). *Two ways to count to ten.* New York: Holt.

Falwell, C. (1993). *Feast for 10.* New York: Clarion Books.

Friedman, A. (1994). *The king's commissioners.* New York: Scholastic.

Gag, W. (1928). *Millions of cats.* New York: Coward-McCann.

Giganti, P. (1988). *How many snails? A counting book.* New York: Greenwillow.

Giganti, P. (1992). *Each orange had 8 slices.* New York: Greenwillow.

Greenfield, E. (1989). *Aaron and Gayla's counting book.* Boston: Houghton Mifflin.

Hoban, T. (1981). *More than one.* New York: Greenwillow.

Hutchins, P. (1986). *The doorbell rang.* New York: Greenwillow.

Jaspersohn, W. (1993). *Cookies.* Old Tappan, NJ: Macmillan.

Juster, N. (1961). *The phantom tollbooth.* New York: Random House.

Linden, A. M. (1994). *One sailing grandma: A Caribbean counting book.* New York: Heinemann.

Lobal, A. (1970). *Frog and toad are friends.* New York: Harper-Collins.

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McKissack, P. C. (1992). *A million fish...more or less.* New York: Knopf.

Munsch, R. (1987). *Moira's birthday.* Toronto: Annick Press.

Myller, R. (1990). *How big is a foot?* New York: Dell.

Norton, M. (1953). *The borrowers.* New York: Harcourt Brace.

Parker, T. (1984). *In one day.* Boston: Houghton Mifflin.

Pluckrose, H. (1988). *Pattern.* New York: Franklin Watts.

San Souci, R. (1989). *The boy and the ghost.* New York: Simon-Schuster Books.

St. John, G. (1975). *How to count like a Martian.* New York: Walck.

Schwartz, D. (1985). *How much is a million?* New York: Lothrop, Lee, & Shepard.

Sharmat, M. W. (1979). *The 329th friend.* New York: Four Winds Press.

Tahan, M. (1993). *The man who counted. A collection of mathematical adventures.* New York: Norton.

Wells, R. E. (1993). *Is the blue whale the biggest thing there is?* Morton Grove, IL: Whitman.

Wolkstein, D. (1972). *8,000 stones.* New York: Doubleday.